View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Elsevier - Publisher Connector

Discrete Applied Mathematics 18 (1987) 175–184
North-Holland

HIERARCHICAL APPROACH TO THE PROCESS PLANNING PROBLEM

Andrew KUSIAK
University of Manitoba, Winnipeg, Canada

Gerd FINKE
Technical University of Nova Scotia, Halifax, Canada

Received 6 March 1987

A hierarchical approach to the process planning problem in manufacturing systems is presented. The model developed consists of the following three subproblems: (1) the tool path selection, (2) the tool path sequencing and (3) the process selection. These problems lead to three distinct combinatorial optimization problems which are characterized and for which solution procedures are discussed.

1. Introduction

In manufacturing systems, one has the following three basic optimization problems:
(1) the design problem,
(2) the operational problem, and
(3) the process planning problem.
The design problem deals with the selection of manufacturing components such as machines, robots, storage systems, etc. (Kusiak [9]). The operational problem is to ensure an optimal utilization of manufacturing components. These two problems are linked by the process planning problem. The solution to the process planning problem has an impact on the formulation of the operational problem and also the solution to the design problem influences the solution to the process planning problem. Due to the high capital involvement, these three problems are of particular importance in an automated form of a manufacturing system, known as a Flexible Manufacturing System (FMS).

In this paper, the process planning problem will be analysed. There are relatively few quantitative approaches to this problem. Bjorke and Haugrud [1] applied topology and graph theory to describe parts. Their approach can be considered as a first step leading to a formulation of the process planning problem. Halevi [6] and Chang and Wysk [3] presented frameworks for the solution of the problem and indicated the need to model the various stages. Yellowley and Kusiak [12] applied a set partitioning formulation to the process planning problem.

0166-218X/87/$3.50 © 1987, Elsevier Science Publishers B.V. (North-Holland)

Instead of formulating the entire process planning problem, we want to present a hierarchical approach. The following three levels are considered:

Level 1: the tool path selection problem,

Level 2: the tool path sequencing problem, and

Level 3: the process selection problem.

This three level method has two advantages: the resulting three subproblems (at levels 1, 2, 3) are much easier to solve than the original problem; and the planning process and the human operators may interact more easily.

2. The tool path selecton problem (P1)

2.1. Problem illustration

In order to formulate problem P1, consider the part in Fig. 1 with a set $\{v_1, v_2, v_3, v_4\}$ of material volumes to be removed. A natural question arises: how to generate these volumes? The problem of finding the depth d_i of a volume v_i has been solved. It is known in the manufacturing and mathematical programming literature as the depth of the cut problem (see for example Hitomi [7] and Philipson and Ravindran [10]). The length l_i of a volume v_i is determined by two adjacent edges and can be generated by a computer-aided design system. Based on the solution to the depth of the cut problem and knowing the length l_i of a volume v_i, a volume removal cost can be calculated (Yellowley and Kusiak [12]).

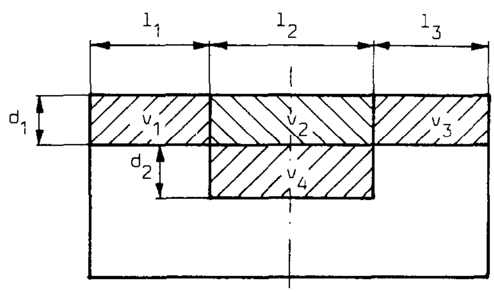

Fig. 1. A three-dimensional part with a set $\{v_1, v_2, v_3, v_4\}$ of material volumes to be removed.

For the part in Fig. 1, one can construct the incidence matrix (1).

$$
\begin{array}{c}
\text{feasible tool path} \\
\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array}
\end{array}
$$

$$
A = [a_{ij}] = \left[\begin{array}{ccccccc}
1 & & & & 1 & & 1 \\
& 1 & & & 1 & 1 & \\
& & 1 & & 1 & & 1 \\
& & & 1 & & 1 &
\end{array}\right]
\begin{array}{l} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}
\quad \begin{array}{l}\text{material} \\ \text{volume to be} \\ \text{removed}\end{array} \qquad (1)
$$

$$
\begin{array}{ccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 \end{array}
$$

Each row i of the above incidence matrix A represents a material volume to be removed. Each column j of (1) corresponds to a feasible tool path at cost c_j. For example, in the tool path 5 (column $j=5$) volumes v_1, v_2 and v_3 are removed at cost c_5.

Let p_j denote a tool path corresponding to column j. A process plan P is a set of tool paths $\{p_{j_1}, p_{j_2}, \dots, p_{j_r}\}$ such that each volume is removed at least once. An optimal process plan P^* is a process plan with the minimum corresponding cost.

2.2. *Integer programming formulation*

Let us introduce the following notation:

I set of indexes for volumes to be removed.
J set of feasible tool paths.
$a_{ij} = \begin{cases} 1 & \text{if volume } v_i \text{ is removed in tool path } j, \\ 0 & \text{otherwise.} \end{cases}$
T set of available tools for machining a given part.
N upper limit on a number of tools to be used for machining a given part.
J_t subset of tool paths for which tool $t \in T$ has been used ($\bigcup_{t \in T} J_t = J$).
c_j cost of tool path j, $j \in J$.
$\tilde{p}_t$ utilization cost of a tool t, $t \in T$.
$x_j = \begin{cases} 1 & \text{if tool path } j \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases}$
$y_t = \begin{cases} 1 & \text{if tool } t \text{ is selected, } t \in T, \\ 0 & \text{otherwise.} \end{cases}$

The objective is to minimize the total sum of tool path costs and tool utilization costs. Hence we obtain the following formulation.

(P1)
$$Z = \min \sum_{j \in J} c_j x_j + \sum_{t \in T} \tilde{p}_t y_t \tag{2}$$

$$\text{s.t.} \sum_{j \in J} a_{ij} x_j \geq 1 \qquad \text{for all } i \in I, \tag{3}$$

$$\sum_{t \in T} y_t \leq N \tag{4}$$

$$\sum_{j \in J_t} x_j \leq |J_t| \, y_t \qquad \text{for all } t \in T, \tag{5}$$

$$x_j = 0, 1 \qquad \text{for all } j \in J, \tag{6}$$

$$y_t = 0, 1 \qquad \text{for all } t \in T. \tag{7}$$

Here $|J_t|$ indicates the cardinality of the set J_t. Constraint (3) ensures that each material volume is removed in at least one tool path. Inequality (4) imposes an upper limit on the number of tools to be used for machining a given part. Constraint (5) implies that a tool path only can be made if the corresponding tool has been used.

2.3. Solution to problem P1

The structure of problem P1 is well suited for a Lagrangean relaxation. Readers not familiar with this technique may refer to recent overview papers, i.e., Fisher [5] and Shapiro [11]. The Lagrangean relaxation approach has the following two advantages: a feasible solution is available at every iteration of this algorithm, and a distance between the feasible solution and the optimal solution can be estimated.

Dualizing on constraint (5) of problem P1 yields the following relaxed problem

$$\text{(R)} \qquad \begin{aligned} Z_{\mathrm{D}}(u_t) &= \min \sum_{j\in J} c_j x_j + \sum_{t\in T} u_t\Big(\sum_{j\in J_t} x_j - |J_t|\, y_t\Big) + \sum_{t\in T} \tilde{p}_t y_t \\ &= \min \sum_{t\in T}\sum_{j\in J_t} (c_j + u_t)x_j - \sum_{t\in T} (u_t\,|J_t| - \tilde{p}_t) y_t \\ &\quad \text{subject to (3), (4), (6) and (7).} \end{aligned}$$

This relaxed problem is defined for $u_t \geq 0$, $t\in T$, which is a necessary condition for $Z_{\mathrm{D}}(u_t) \leq Z$. The best choice for u_t is the optimal solution to the dual problem

$$\text{(D)} \qquad Z_{\mathrm{D}} = \max_{u_t} Z_{\mathrm{D}}(u_t) \quad \text{subject to (3), (4), (6) and (7).}$$

To solve problem P1 the following framework of the Lagrangean relaxation algorithm has been applied:

Step 0. Set iteration number $k=1$ and choose initial values for the Lagrangean multipliers u_t^k, $t\in T$.

Step 1. Solve problem R for u_t^k, $t\in T$. The value obtained $Z_{\mathrm{D}}(u_t^k)$ is a lower bound (LB) on the value of the objective function Z_{D} in (D).

Step 2. Generate a feasible solution to problem P1. The value Z of P1 is an upper bound (UB) on the value of the objective function Z_{D} of (D).

Step 3. If the current solution to the problem D satisfies a stopping criterion, stop; otherwise update u_t^k, set $k=k+1$ and go to Step 1.

Below, a discussion of this algorithm is given. The algorithm performed well for initial values $u_t^1 = \max_{j\in J_t} c_j$ in Step 0.

Problem R (Step 1) decomposes into two subproblems:

(1) the set covering problem and

(2) the trivial knapsack problem.

To solve the set covering problem, the heuristic proposed by Chvatal [4] has been applied. Solution of the second subproblem is based on selecting the first N smallest values $u_t\,|J_t| - \tilde{p}_t$.

For Step 2, a modified heuristic based on Chvatal's algorithm [4] was proposed. The original heuristic selects tool paths (columns) of matrix A based on a ratio $c_j/|I_j|$, where $|I_j|$ is the number of 1's in column j. The selected tool paths correspond to the subscripts of the first $|J_c|$ smallest values $c_j/|I_j|$, where J_c is a set of tool paths in A such that each volume v_i is removed at least once. The solution generated in this manner does not always satisfy constraint (7). To satisfy this constraint, set J_c has to be modified by eliminating those tool paths which correspond to a single tool in set J_c, then two tools in set J_c, etc. The removed tool paths are replaced by those which require a smaller number of tools.

As the stopping criterion (Step 3), the following rule was applied:

$$(\mathrm{UB}-\mathrm{LB})/\mathrm{LB} \leq \varepsilon$$

where ε is a small positive number.

This procedure has been used to solve randomly generated problems. Some computational results for random matrices $[a_{ij}]$ of size $|I| \times |J|$ with density 0.2 are given in Table 1.

Table 1
Computational results for problem P1 solved on a CDC CYBER 170-720.

$\lvert I \rvert$	$\lvert J \rvert$	Number of problems solved	ε%	Average number of iterations	Average CPU time (secs)
10	100	6	5%	12	4.0
20	100	6	5%	11	12.28
20	200	6	5%	14	23.10

3. The tool path sequencing problem (P2)

The solution to the problem P1 is a set of tool paths. For the part in Fig. 1 assume that the following tool paths have been generated

$$p_1 = \{v_1\}, \qquad p_2 = \{v_2, v_3\}, \qquad p_3 = \{v_4\}.$$

The volume v_4 is below volume v_2. Consequently, v_4 cannot be removed before v_2. The arrangement of the volumes imposes precedence constraints on the tool paths p_1, p_2, p_3 which can be represented by the following digraph

$$\begin{tikzcd} p_1 \arrow[dr] & \\ & p_3 \\ p_2 \arrow[ur] & \end{tikzcd}$$

where $p_i \rightarrow p_j$ means that the tool path p_i precedes the tool path p_j. For this digraph, one obtains the two feasible process plans:

$$\{p_1, p_2, p_3\} \quad \text{and} \quad \{p_2, p_1, p_3\}.$$

In general, problem P2 is equivalent to the single machine sequencing problem with precedence constraints. One may solve P2 by the topological sorting algorithm. A variant of this method can be found in Horowitz and Sahni [8]. If the precedence digraph consists of $n=|J|$ vertices (representing the tool paths p_j) and e edges (precedence relations), then efficient implementations result in a computational time complexity of $O(n+e)$. Hence the tool path sequencing problem P2 is an easy combinatorial problem which can be solved in linear time with respect to its size.

4. The process selection problem (P3)

4.1. Detailed definition of process plans

The two previous problems (P1 and P2) are applicable to classical manufacturing systems and flexible manufacturing systems. The process selection problem applies exclusively to FMSs. One of the features of an FMS is a routing flexibility which is measured by the number of different routes (process plans) in which a part can be manufactured (Buzacott [2]). To ensure the high ratio of routing flexibility for each part, one may generate different process plans (i.e., solutions to P2), each of which is characterized by its specific tool and fixture requirement.

Let us augment the definition of a process plan. Define the processs plan P_i as a set of 3-tupes: $P_i=\{(V_1;t_1,f_1),\dots,(V_m;t_m,f_m)\}$, where $V_l=(v_{l_1},\dots,v_{l_n})$ is a set of material volumes to be removed with a given tool and without changing the fixture. V is the set of all volumes to be removed for a given part (note that $V=\bigcup_{l=1}^{m} V_l$ where m denotes the total number of tools and fixtures). Let t_l be a tool for removing V_l and f_l be a fixture for presenting a part while removing V_l.

To illustrate this definition, consider the three-dimensional part in Fig. 2 with material volumes $v_1, v_2, \dots, v_9$ to be removed.

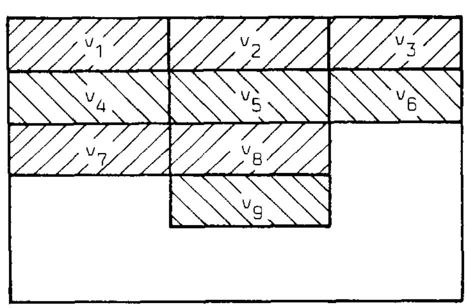

Fig. 2. A part with a set $\{v_1, v_2, \dots, v_9\}$ of volumes to be removed.

For the part in Fig. 2, one may have the following two process plans:

$$P_1=\{(v_1,v_2,v_3;t_1,f_1),(v_4,v_5,v_6;t_1,f_1),(v_7,v_8;t_2,f_1),(v_9;t_3,f_2)\},$$

$$P_2 = \{(v_2, v_5, v_8, v_9;\ t_4, f_2), (v_1, v_4, v_7;\ t_5, f_3), (v_3, v_6;\ t_6, f_3)\}.$$

The corresponding costs of removing material volumes $v_1, v_2, \ldots, v_9$ are c_1 and c_2, where usually $c_1 \neq c_2$.

Assign to each process plan P_i, $i \in N = \{1, 2, \ldots, n\}$, the following incidence vector

$$x_i = [x_{i1}, \ldots, x_{ia}, x_{ib}, \ldots, x_{ic}]$$

where

$$x_{it} = \begin{cases} 1 & \text{if tool } t \text{ has to be used for } P_i,\ t \in \{1, \ldots, a\}, \\ 0 & \text{otherwise,} \end{cases}$$

$$x_{if} = \begin{cases} 1 & \text{if fixture } f \text{ has to be used for } P_i,\ f \in \{b, \ldots, c\}, \\ 0 & \text{otherwise.} \end{cases}$$

For any two process plans, P_i and P_j, define the weighted Hamming distance

$$d_{ij} = \sum_{k=1}^{c} w_k \delta(x_{ik}, x_{jk}) \quad \text{for all } i, j$$

where

$$\delta(x_{ik}, x_{jk}) = \begin{cases} 1 & \text{if } x_{ik} \neq x_{jk}, \\ 0 & \text{otherwise} \end{cases}$$

and w_k is the weight coefficient of attribute k.

We have modified the Hamming distance by introducing the weight coefficient w_k for each attribute k. This is due to different importance of each attribute. For example, weights assigned to fixtures will typically have much higher values than weights assigned to tools.

4.2. Formulation of the process selection problem (P3)

Let $K = \{1, 2, \ldots, m\}$ denote the set of parts that are to be manufactured. For each part $k \in K$ a set of possible process plans N_k is available. Let $N = \bigcup N_k$ indicate the set of all process plans.

Consider the complete m-partite graph $G = (N, A)$ with the process plans as nodes. Let the set A consist of all (non-directed) arcs connecting the elements of N_k and N_l, for all pairs $k, l \in K$ with $k \neq l$. In addition, we associate with each node or process plan P_i its cost c_i and with each arc the weighted Hamming distance d_{ij}.

The process selection problem P3 considers all sets $S = \{S_1, S_2, \ldots, S_m\}$ that contain one representative node S_k from each N_k. This ensures that each part can be manufactured. Define $A^s \subseteq A$ as the set of arcs between the points of S and define the total cost as follows

$$\sum_{(i,j) \in A^s} d_{ij} + \sum_{i \in S} c_i.$$

Then problem P3 is to determine the maximum clique (S, A^s) with minimal cost.

The given cost function seems to be an appropriate measure since it will select low cost process plans which require globally the smallest set of tools and fixtures.

Problem P3 may also be formulated as an integer programming problem. Introduce the variables

$$y_i = \begin{cases} 1 & \text{if process plan } i \text{ is selected,} \\ 0 & \text{otherwise,} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if process plans } i \text{ and } j \text{ are selected,} \\ 0 & \text{otherwise.} \end{cases}$$

Then one obtains P3 in the following form

$$\min \tfrac{1}{2} \sum_{(i,j)\in A} d_{ij} y_{ij} + \sum_{i \in N} c_i y_i \tag{8}$$

$$\text{s.t.} \quad \sum_{i \in N_k} y_i = 1 \qquad \text{for all } k \in K, \tag{9}$$

$$y_i + y_j - 1 \le y_{ij} \qquad \text{for all } (i,j) \in A, \tag{10}$$

$$y_i = 0, 1 \qquad \text{for all } i \in N, \tag{11}$$

$$y_{ij} = 0, 1 \qquad \text{for all } (i,j) \in A. \tag{12}$$

The factor $\frac{1}{2}$ is included in (8) since the term $d_{ij} y_{ij} = d_{ij} y_{ji}$ appears twice. Note also that the integrality condition (12) is redundant.

4.3. Solution procedures for problem P3

Large scale process selection problems cannot be solved exactly. Also the linear relaxation of the integer program (8)–(12) is not very useful. A very efficient heuristic is the following

(i) Generate a random permutation $\pi(k)$, $k \in K$, for the set $K = \{1, 2, \dots, m\}$ of parts.

(ii) *Construction Step.* A set S of representatives is selected in the order of π, i.e., first $S_{\pi(1)} \in N_{\pi(1)}$, then $S_{\pi(2)} \in N_{\pi(2)}, \dots,$ and finally $S_{\pi(m)} \in N_{\pi(m)}$. The selection is done as follows: Start with the empty set $S = \emptyset$ and for $k = 1, 2, \dots, m$ select $j = S_{\pi(k)} \in N_{\pi(k)}$ which minimizes $c_j + \sum_{s \in S} d_{sj}$ and augment $S \leftarrow S + \{S_{\pi(k)}\}$.

(iii) *Exchange Step.* For $k = 1, 2, \dots, m$ compute

$$\min_{j \in N_{\pi(k)}} \left\{ c_j + \sum_{\substack{s \in S \\ s \ne N_{\pi(k)}}} d_{sj} \right\}$$

and if the minimum is attained for an index $j_M \ne S_{\pi(k)}$, exchange this pair $S \leftarrow S - \{S_{\pi(k)}\} + \{j_M\}$. Then repeat the full loop whenever at least one exchange has taken place.

As illustration, consider the following problem with three parts $K=\{1,2,3\}$, the sets of process plans $N_1=\{1,2\}$, $N_2=\{3,4\}$, $N_3=\{5,6,7,8\}$, and the incidence matrix

Process Plans

$$
\begin{array}{lc}
 & \begin{array}{cccccccc} \multicolumn{2}{c}{\overbrace{\text{Part 1}}} & \multicolumn{2}{c}{\overbrace{\text{Part 2}}} & \multicolumn{4}{c}{\overbrace{\qquad\text{Part 3}\qquad}} \\ 1&2&3&4&5&6&7&8 \end{array} \\
\begin{array}{r} \text{Tools}\left\{\begin{array}{l} t_1\\ t_2\\ t_3\\ t_4\end{array}\right. \\ \text{Fixtures}\left\{\begin{array}{l} f_1\\ f_2\\ f_3\end{array}\right.\end{array} & \left[\begin{array}{cccccccc}
1 & & 1 & 1 & & 1 & 1 & 1 \\
1 & 1 & & 1 & 1 & 1 & 1 & \\
 & 1 & 1 & 1 & 1 & & 1 & 1 \\
1 & & & 1 & 1 & 1 & & 1 \\
1 & & 1 & 1 & 1 & & 1 & \\
 & 1 & 1 & 1 & 1 & 1 & & 1 \\
1 & & & & 1 & 1 & 1 & 1
\end{array}\right]
\end{array}
$$

Let the cost vector for the process plans be

$$c=(7.5, 8.1, 9.4, 11.6, 5.4, 7.3, 6.2, 5.9)$$

and let the weight vector for the tools and fixtures be

$$w=(1,1,1,1,1,1,1)$$

Then the following matrix $D=(d_{ij})$ of Hamming distances is obtained

$$
\begin{bmatrix}
- & - & 5 & 3 & 3 & 2 & 2 & 4 \\
- & - & 3 & 3 & 3 & 4 & 4 & 4 \\
5 & 3 & - & - & 4 & 5 & 3 & 3 \\
3 & 3 & - & - & 2 & 3 & 3 & 3 \\
3 & 3 & 4 & 2 & - & - & - & - \\
2 & 4 & 5 & 3 & - & - & - & - \\
2 & 4 & 3 & 3 & - & - & - & - \\
4 & 4 & 3 & 3 & - & - & - & -
\end{bmatrix}
$$

One has a total number of $|N_1|\times|N_2|\times\cdots\times|N_m|=2\times2\times4=16$ different selection sets S. The heuristic (Steps (i)–(iii)) may be started with any of the $|K|=m!=6$ permutations. The optimal solution for the example is

$$S^*=\{P_1, P_4, P_5\}$$

with the total cost 32.5. One may verify that this solution is obtained if the heuristic is initiated with the permutation $\pi(1)=3$, $\pi(2)=1$, $\pi(3)=2$. The other permutations generate two local minima with values 32.9 and 33.1 which represent in fact the second and third best solutions.

For a general problem, the heuristic should be performed repeatedly in order to avoid poor local minima. However, only few reruns are required (less than 10) since usually the best solution occurs repeatedly at an early stage. In addition, the heuristic is fast, e.g., one full run of a random problem with 100 process plans and 25 parts takes about 0.4 sec on a CDC CYBER 170-720. In all our test cases, the generated solutions seem to be optimal since they could not be improved by even a large number of reruns.

References

[1] O. Bjorke and B. Haugrud, Mathematical methods in planning of machining operations, Paper presented at the CIRP Meeting, Ann Arbor, MI, 1967.

[2] J.A. Buzacott, The fundamental principles of flexibility in manufacturing systems, Proc. First International Conference on Flexible Manufacturing Systems, Brighton, U.K. (1983) 13-22.

[3] T.C. Chang and R. Wysk, An Introduction to Automated Process Planning Systems (Prentice-Hall, Englewood Cliffs, NJ, 1985).

[4] V. Chvatal, A greedy heuristic for the set covering problem, Math. Op. Res. 4 (1979) 223-235.

[5] M.L. Fisher, The Lagrangean relaxation method for solving integer programming problems, Management Sci. 27 (1981) 1-18.

[6] G. Halevi, The Role of Computers in Manufacturing Processes (Wiley, New York, 1980).

[7] K. Hitomi, Manufacturing Systems Engineering (Taylor and Francis, London, 1979).

[8] E. Horowitz and S. Sahni, Fundamentals of Data Structures in Pascal (Computer Science Press, Rockville, MD, 1984).

[9] A. Kusiak, Application of operational research models and techniques in flexible manufacturing systems, Europ. J. Oper. Res. 4 (3) (1986) 336-345.

[10] R.H. Philipson and A. Ravindran, Application of mathematical programming to metal cutting, Math. Programming 11 (1979) 116-134.

[11] J.F. Shapiro, A survey of Lagrangean techniques for discrete optimization, Annals Discrete Math. 5 (1979) 113-138.

[12] I. Yellowley and A. Kusiak, Observations on the use of computers in the process planning of machined components, CSME Trans. 9 (1985) 70-74.